20 Pricing under network effects
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Abstract

Pricing in markets characterized by network effects is a topic that has recently been attracting considerable interest from researchers in both marketing and economics. Early literature on static pricing under network effects focused on the importance of consumer expectations and the multiple equilibria problem. In a dynamic setting, penetration pricing has been found to be optimal under various scenarios. After reviewing the analytical literature on pricing under network effects, we discuss its connections to other literatures. Empirical studies have been relatively scarce. One obstacle is the computational burden in solving for the optimal pricing policies. We illustrate the issues involved in empirical studies and suggest directions for future research.

Network effects arise when the utility of an agent from consumption of a good increases with the number of other agents consuming the same good. A classic example is communication networks – telephones, fax machines, or e-mail accounts become more valuable as more people join the network, i.e. adopt the product.

Network effects can be direct or indirect. Under direct network effects, the utility that a consumer derives from a good depends directly on its installed base, or equivalently the cumulative unit sales of the good. The communication networks mentioned above are examples of direct network effects. They are in contrast with indirect network effects, under which consumers care about the installed base only because a large installed base of the good will increase the availability of a complementary good. For example, a person purchasing a video game console will be concerned with the number of other people purchasing the same hardware because a more popular game console will induce more games to be developed for it. Such a hardware–software paradigm applies to many other industries such as compact disks (CDs), digital video disks (DVDs), personal computers (PCs), personal digital assistants (PDAs), video cassette recorders (VCRs) and so on.

A wide range of industries are characterized by network effects. Some of these network effects may appear in more subtle ways. For example, more people going to a shopping mall can make it more crowded. On the other hand, a more popular shopping mall may attract more and better-quality stores. If the second effect dominates, the utility of going to the shopping mall increases with the number of people going there, which gives rise to the indirect network effect. In the case of QWERTY keyboards, there is a direct effect because people like to be able to type on others' keyboards. There may also be an indirect effect because the dominant keyboard design will draw more compatible products and services.

Network effects add interesting dimensions to firms' strategies: should the new product generation be compatible with the old one? Should the new system standard be proprietary or open to other firms? But pricing continues to be a critical element for firms that compete in these markets. In the following sections, we discuss the issues that require special attention for pricing under network effects.

The rest of this chapter is organized as follows. We first introduce the issues involved in static pricing, dynamic pricing and nonlinear pricing under network effects. Then we

compare such pricing issues with those in other areas such as two-sided markets, switching costs and economies of scale. Once we have a clear picture of the analytical literature, we proceed to discuss empirical studies and provide an illustrative example. Finally, we conclude and suggest directions for future research.

1.   Static pricing

We start from simple static pricing in a monopoly market, which introduces the important issues of consumer expectations and multiple equilibria. Rohlfs (1974) provides an early treatment of such issues in the context of a communication network, although the fulfilled-expectations demand curve has been discussed in Leibenstein (1950). We discuss them below.

Consumer expectations play an important role in the adoption of network products. At the time of making purchase decisions, consumers do not know exactly how many people will adopt the product. Such information is needed while making purchase decisions since a consumer's utility from the product depends on the network size. Therefore consumers' purchase decisions are based on the expected size of the network.

One commonly proposed restriction to be placed on expectations is that they will be fulfilled in the sense that consumer expectations are consistent with the actual outcome in the market (see, e.g., Leibenstein, 1950; Rohlfs, 1974; Katz and Shapiro, 1985; Economides, 1996). That is, on the induced fulfilled-expectations demand curve, each price p corresponds to those quantities q such that, when consumers expect quantity q, there will be just q consumers purchasing at price p. Leibenstein (1950) derives such a demand curve from fixed-expectations demand curves. Assume a fixed-expectations demand curve $q=D^x(p)$ if all consumers believe the total demand is x. Varying x will result in a set of fixed-expectations demand curves. On each $D^x(p)$, there is a point where the actual demand is consistent with consumers' expectations, i.e. $x=D^x(p)$. As illustrated in Figure 20.1, the locus of all these points forms the fulfilled-expectations demand curve $D(p)$. Leibenstein argues that such a demand curve is more elastic than any of the fixed-expectations demand curves from which it is derived.

Multiple equilibria may occur even if we restrict attention to fulfilled expectations. Intuitively, if each consumer believes that no other consumer buys the network product, then it may result in the case that no one will buy it, which leads to a fulfilled-expectations equilibrium with zero sales. However, if each consumer expects many others to purchase the product, then many people will purchase, and this outcome is another fulfilled-expectations equilibrium.

Multiple equilibria show up graphically as multiple intersections between the fulfilled-expectations demand curve and the horizontal line corresponding to a given price level. Implicitly this means that the demand curve has both upward-sloping segments and downward-sloping ones. For reasons explained by Rohlfs (1974), the equilibria located on the upward-sloping segments may be ruled out because they are unstable. However, there could still be multiple equilibria which are stable, and hence the exact demand at any given price level has to be determined carefully.

If multiple equilibria are possible, firms will try to affect consumer expectations so that the largest equilibrium quantity can be achieved at a given price level. Shapiro and Varian (1998) discuss various tactics in managing consumer expectations. In particular, a low introductory price, or penetration pricing, can help convince consumers that the
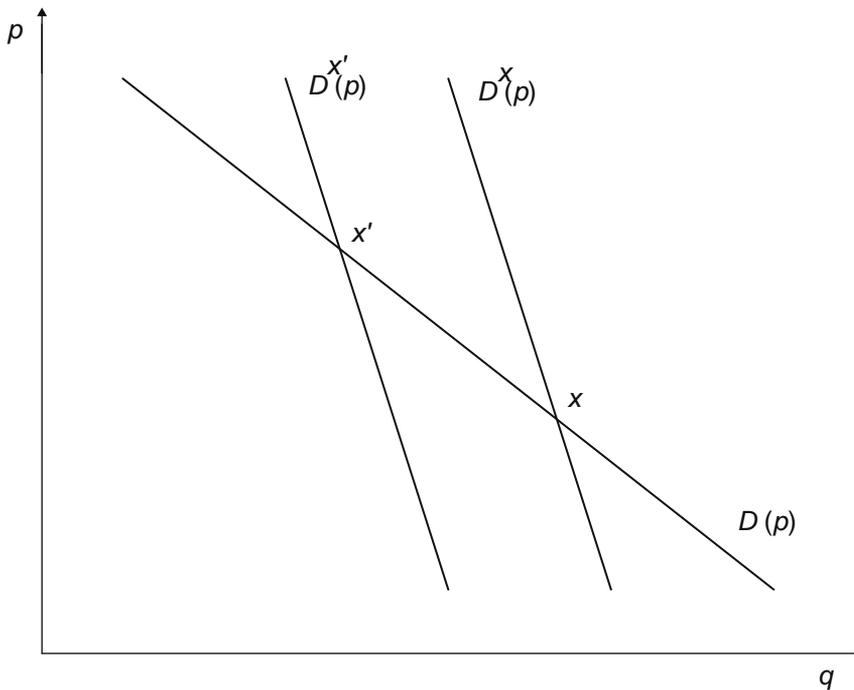
Source: Adapted from Leibenstein (1950), Figure 1.

Figure 20.1   Fulfilled-expectations demand curve

product will be successful in the future. Further discussion on penetration pricing follows in the next section.

2.   Dynamic pricing

The diffusion of a network product takes place over time. During the life cycle of the product, firms may want to charge different prices according to evolving market conditions. Thus firms' pricing strategies can be better captured through a dynamic model.

When a network product is just launched, it may not be very attractive to consumers because of its limited network size. This provides an incentive for the firm to set a low initial price in order to encourage consumer adoptions. Once many consumers have joined the network and hence the product has become more attractive, the price can be raised. This low-high pricing scheme is often referred to as penetration pricing.

According to Cabral et al. (1999), the early telephone network provides a good example of penetration pricing. Bell's 1876 patents created a monopoly over the telephone service until the expiration of these patents in 1893. In this period, average monthly fees charged by the unregulated telephone companies rose steadily.

Monopoly pricing

In a monopoly market for durable goods, firms' incentives for penetration pricing are in contrast with the Coase conjecture (Coase, 1972). Coase (1972) argues that durable-goods

monopolists have incentives to keep cutting prices in order to further penetrate the market. Anticipating this, forward-looking consumers will delay purchases until prices equal marginal costs. Therefore, unless there is a way for these monopolists to credibly commit to future prices, they will not be able to exercise any market power. But under network effects, if indeed a monopolist finds it optimal to engage in penetration pricing and as a result prices keep rising, the Coasian dynamics (Hart and Tirole, 1988) may no longer be applicable.

Bensaid and Lesne (1996) study the optimal pricing policy of a monopolist selling a durable good. They start with a two-period model and then extend it to an infinite number of periods. In each period the network benefit is assumed to be proportional to the previous installed base. They find equilibrium prices to be increasing over time when the network effect is of sufficient magnitude.

Using a two-period model, Cabral et al. (1999) study when and why a monopolist would set a low introductory price. They find that, when consumers are price-takers, Coasian dynamics tend to predominate over penetration pricing if there is complete information. Penetration pricing occurs when each consumer's valuation of the product is her private information, or when consumers are not perfectly informed about the firm's unit cost.

Mason (2000) develops a continuous-time, infinite-horizon model in which a monopolist chooses output to maximize the present value of profits from production of a durable good. Consumers decide whether to adopt according to the current price and the expected network benefit. Under this configuration they show that the monopolist prices at marginal cost, as predicted by Coase (1972).

Gabszewicz and Garcia (2005) solve explicitly for the optimal price path in a monopoly market with a finite number of time periods. A somewhat unusual feature in their framework is that consumers are 'short-lived' in the sense that there is a different cohort of consumers making purchase decisions in each new time period. They find an increasing price path, i.e. penetration-like pricing, to be optimal.

Competition

Many papers on dynamic pricing under network effects have focused on monopoly markets. Dealing with competition in the market adds to the complexities in solving for firms' optimal policies. In general, the incentives for penetration pricing still exist in oligopoly markets. However, one difference is that competition would limit the market power of each firm.

If penetration pricing does occur, competition might push initial prices to be even lower than those under monopoly. But on the other hand, there is splintering of the market under oligopoly but not under monopoly. Thus a monopolist may expect more profits in the second period than oligopolists, and hence may be willing to cut initial prices even lower. Therefore it is unclear whether monopoly or oligopoly leads to lower initial prices.

Katz and Shapiro (1986) study the adoption pattern of competing technologies depending on whether these technologies are sponsored or not. If a technology is sponsored, an entity owns property rights to the technology and hence is willing to make investments to promote it. In the absence of a sponsor, free entry into the supply of a technology will lead to marginal cost pricing. Katz and Shapiro consider two periods or generations of

homogeneous consumers, and two incompatible technologies. In each period consumers choose to adopt one of the two standards. If both standards are sponsored, they find that the firm with the superior standard in the second period may decide to price below cost in the first period in order to attract consumers to join its network.

Xie and Sirbu (1995) model the dynamic pricing behaviors of an incumbent and a later entrant. They incorporate network effects into a diffusion model with finite horizon and continuous time. The dynamic potential demand depends on the current network sizes. They establish optimal pricing policies with open-loop controls, i.e. firms set a one-shot price trajectory without feedback effects. This is as opposed to closed-loop controls, in which case firms set a state-contingent pricing policy and adjust for any changes in market conditions. Xie and Sirbu show that, under strong network effects, an increasing price path can be optimal. Also, with strong network effects and a small installed base, the incumbent profits from a compatible entry.

Nondurable goods
The aforementioned studies concentrate on durable-goods markets, in which a consumer will drop out of the market after making a purchase. In these markets a consumer's utility is affected by the cumulative sales of the durable product. This may not be true for nondurable-goods markets.

For example, consider Xbox Live, a subscription service offered by Microsoft for online gaming. In each month, some consumers may join or drop out of the network. So the subscription level fluctuates over time. When a potential customer decides whether to subscribe to the service, she cares about how many people she can play with, i.e. the current subscription level.

If consumers' utilities are affected by the current subscription level, not historical levels, then it seems that there is no intertemporal price effect, and the producer can set prices to maximize single-period profits only. However, past prices or quantities may affect current demand through consumer expectations or usage experiences. Therefore the producer's pricing problem may still be dynamic, and it turns out that an increasing price path can be optimal for nondurable goods as well.

Dhebar and Oren (1985) analyze a monopolist's intertemporal pricing decision for a new subscription service. In each time period all consumers decide whether to subscribe based on the previous level of subscription and their anticipation about the network growth. The potential demand is defined as $d^\alpha(x, p)$, where x is the previous subscription level and p is the price. $\alpha \in [0, 1]$ governs consumer expectations on network growth. $\alpha = 1$ indicates that consumers have rational expectations and $\alpha = 0$ indicates that consumers are myopic and base their subscription decisions on the previous subscription level only.

The monopolist sets a price trajectory $p(t)$ by solving the following optimization problem:

$$\underset{p(t)}{\text{Max}} \int_0^\infty e^{-\delta t}[px - c(x)]dt$$

subject to

$$x(0) = x_0,$$
$$x'(t) = G(d^\alpha, x)$$

Here $G(d^\alpha, x)$ describes the product diffusion process. Standard control theory is then applied to solve for the optimal price trajectory. They demonstrate that typically the price path is increasing and the firm may set initial prices below marginal costs. It is also shown that higher growth anticipations and a lower discount rate result in a lower equilibrium price and a larger network.

Consumer expectations
Under network effects, consumers' adoption decisions critically depend on their expectations on future network sizes. The assumption of fulfilled expectations or rational expectations indicates that consumers can perfectly predict the future network sizes if there is perfect information and no uncertainty, and when there is imperfect information or uncertainty, consumers can use all available information to make the best possible predictions. This might require too much faith in consumers' cognitive processing power.

In dynamic settings, firms are forward looking in the sense that they maximize the present discounted value of total profits over a planning horizon. Regarding consumer adoption decisions, however, past studies have made various assumptions ranging from completely myopic to perfectly rational.

For example, Xie and Sirbu (1995) assume myopic consumers in the sense that consumers' adoption decisions are based on the current prices and network sizes, not the expected future ones.

Bensaid and Lesne (1996) assume that the value of the product is a function of the existing network size, but consumers still form rational expectations about the future network size in order to decide when to purchase the product.

In the Dhebar and Oren (1985) model, a consumer's adoption decision depends on the expected network size. However, fulfilled expectations are not enforced. Instead, the expected network size is allowed to vary between the existing network size and fulfilled future network size.

Radner and Sundararajan (2005) examine how the predictions would change if the assumption of unbounded rationality were relaxed in a monopoly market for a subscription service. They assume that consumers are boundedly rational in two aspects. First, not all consumers observe a price change immediately. Only a fraction of consumers respond to new prices, while others make no adjustment. Second, consumers are not able to make accurate forecasts on future demand. In particular, they examine a model with myopic consumers and then extend it to other cases.

They use a continuous-time, infinite-horizon model to study the dynamic pricing problem of a network monopolist. They find that the price is zero when the product user base is below a specific threshold. Once this threshold is crossed, the price is chosen to keep user base stationary. They show that this pricing policy is robust to several alternative models of bounded rationality.

3.   Nonlinear pricing
So far we have restricted our attention to those markets in which each consumer buys at most one unit of the network good. In such markets only uniform pricing is relevant. However, in some other markets it may happen that different consumers buy variable quantities of the product. As pointed out by Sundararajan (2003), software purchases from the business segment often fall into this category.

Table 20.1   Analytical studies on pricing under network effects

Pricing Scheme	Market	Paper
Static	Monopoly	Leibenstein (1950)
		Rohlfs (1974)
		Sundararajan (2003)
	Oligopoly	Katz and Shapiro (1985)
		Economides (1996)
Dynamic	Monopoly	Dhebar and Oren (1985)
		Dhebar and Oren (1986)
		Bensaid and Lesne (1996)
		Cabral et al. (1999)
		Mason (2000)
		Gabszewicz and Garcia (2005)
		Radner and Sundararajan (2005)
	Oligopoly	Katz and Shapiro (1986)
		Xie and Sirbu (1995)

For example, the market for PC operating systems exhibits indirect network effects through the availability of compatible software applications. In this market, a company usually buys many copies of an operating system for the computers owned by the company. So the magnitude of the network effect increases with the installed base of an operating system, rather than the total number of buyers. Also, the network benefit to a buyer depends on the quantity she will buy, in addition to the product installed base.

In such scenarios firms have incentives to charge a different price to different quantities. That is, a nonlinear pricing scheme can be designed to extract more consumer surplus and raise profits.

Sundararajan (2003) presents a static model of nonlinear pricing in a monopoly market with fulfilled expectations. It is shown that optimal pricing includes discounts that increase with quantity, and can also involve a two-part tariff. While network effects generally raise prices, consumption may or may not rise.

In the context of a subscription service, Dhebar and Oren (1986) analyze a monopolist's pricing schedule over quantity and time. In each period, a usage-sensitive nonlinear pricing policy is used to induce heterogeneous consumers to self-select different quantities at different marginal prices. Using a numerical example, they show that nonlinear pricing results in a larger equilibrium network because on average it offers consumption access at a lower subscription fee than uniform pricing. Also, nonlinear pricing leads to higher producer surplus, higher total surplus, but smaller consumer surplus than uniform pricing.

Studies on pricing with network effects are summarized in Table 20.1.

4.   Indirect network effects and two-sided markets

In our previous discussion we do not distinguish between two types of network effects, direct network effects and indirect network effects, because the models and the results apply to both. In most studies that we have discussed, consumers' utility functions take the general form of $u(p,x)$, where p is price and x is the installed base of the network

product. This may seem to include direct network effects only, because under indirect network effects, a consumer's utility, $u(p,y)$, depends on y, the availability of the complementary product, and not directly on x, the installed base of the network product itself. However, we can argue that the availability of the complementary product will be a function of the installed base of the network product, i.e.

$$y = f(x)$$

This function f is determined by the market structure for the complementary good. Now the utility function under indirect network effects becomes

$$u'(p, x) = u(p, f(x))$$

which is no different from the general form.

Applying this approach to study the pricing dynamics under indirect network effects, we focus on how a firm would price its network good to consumers, and take the market structure for the complementary good as given. Therefore the two-sidedness of the market is hidden behind the function f that governs the provision of the complementary good.

However, this function f may not be exogenous to the model because it is often the case that a firm has some control over both sides of the market. For example, in the video game industry, console makers (Microsoft, Sony, Nintendo) set the prices of their game consoles (Xbox 360, PS3, Wii), but they also decide the royalties that they charge to the games developed for their consoles. The royalty structure will in turn affect how many games will be provided to each console. Therefore firms may strategically affect the function f through royalty fees, or, more generally, firms may set prices to both sides of the market.

How firms should price to both sides of the market in order to get both sides on board is the central question of a growing literature on two-sided markets. The literature on indirect network effects and the literature on two-sided markets are closely related because conceptually indirect network effects must operate in two-sided markets. The two literatures seem to have different focuses, though. In some sense dynamic pricing under indirect network effects is about a firm's incentive to price-discriminate between early adopters and late adopters, while studies on two-sided markets emphasize a firm's incentive to price-discriminate between two sides of the market.

Indeed, firms often treat two sides of the market asymmetrically. For example, most credit card holders do not have to pay for usage, while merchants are usually charged for each transaction. In contrast, firms that develop PC operating systems adopt the opposite business model. They decide to make money on consumers, not on software application developers. Actually in two-sided markets it is common to see one side pays zero or below cost. Rochet and Tirole (2006) argue that the defining feature of two-sided markets is that the economic outcome is affected by the price structure. In other words, a market is two-sided if the platform can affect the volume of transactions by charging more to one side of the market but reducing the price paid by the other side by the same amount.

A number of studies have examined the market structure in different two-sided markets, e.g. Rochet and Tirole (2002), Schmalensee (2002) on payment cards, Caillaud and

Jullien (2003) on matchmakers, Economides and Katsamakas (2006), Economides and Viard (2006) on operating systems, McCabe and Snyder (2007) on academic journals.

In a general framework, Rochet and Tirole (2003) study how the price allocation between the two sides of the market is affected by a number of factors, including industry structure (monopoly versus duopoly) and governance structure (for-profit versus non-profit). They find that, under both monopoly and duopoly, one side that creates large externalities on the other side will be targeted aggressively by lowering prices. As the number of captive buyers increases, the price to buyers increases while the price to sellers decreases. In the case of competing nonprofit associations, an increase in multi-homing (users access more than one platform) of buyers raises the price to buyers and lowers the price to sellers.

Armstrong (2006) extends the analysis by Rochet and Tirole (2006) and focuses on how the price structure is determined by three main factors: relative size of cross-group externalities, fixed fees or per-transaction charges, single-homing or multi-homing.

Pricing in two-sided markets may look similar to pricing with complementarities. For example, Gillette often sets a low price on its razors but makes money on blades later (Hartmann and Nair, 2007). Nevertheless, there is a subtle difference – in two-sided markets there are complementarities between different customers' consumption decisions.

5.   Relationship to other literatures

Switching costs
Switching costs and network effects are two distinct terms. Switching costs affect a consumer's choice between competing products when she makes repeated purchase decisions. In contrast, the network effect describes the connection between different consumers' purchase decisions on the same product. Farrell and Klemperer (2007) provide a comprehensive survey on the literatures of both switching costs and network effects.

However, there is an analogy between switching costs and network effects. In both cases, early adopters of a product increase the *ex post* market power of its producer. Under switching costs, firms can exercise market power over the same consumers who have been locked in to their products. Under network effects, the market power is over other consumers who have not purchased before.

Therefore, in both cases firms compete *ex ante* for the *ex post* market power, which provides an incentive for penetration pricing. But one difference is that, under switching costs, firms sell to both old and new customers after the first period. If a single price has to be set for both groups of customers, the bargain-then-ripoff incentive might be weakened.

Switching costs and network effects can exist for the same product. We mentioned that the market for the QWERTY keyboard exhibits network effects because a user benefits from a large installed base of the same keyboard design. Additionally there also exist switching costs in this market because it is costly for a user to get used to a different keyboard design.

Doganoglu and Grzybowski (2005) study the dynamic duopoly competition in the presence of both network effects and switching costs, by introducing network effects into the Klemperer (1987) framework of switching costs. Following a Hotelling model, heterogeneous consumers make repeated purchase decisions in two periods. It is assumed

that consumers form rational expectations on future prices and network sizes. They show that stronger network effects imply lower prices in both periods while the impact of switching costs is ambiguous. Also, when network effects are strong and switching costs are moderate, prices in both periods may be lower than those in a market without network effects and switching costs.

Economies of scale

Economies of scale characterize a production process in which the average cost is a decreasing function of the quantity produced. As more consumers adopt a product, the producer may benefit from both economies of scale and network effects, but in different ways. On the production side, economies of scale reduce average costs, while on the demand side, network effects lead to even larger demand. Therefore the network effect is also referred to as demand-side economies of scale.

Due to their similarities, one may expect economies of scale and network effects to have similar implications for firms' pricing policies. Actually this may or may not be true depending on the sources of the scale economies.

Economies of scale tend to occur in industries with high upfront fixed costs, and such fixed costs will be distributed across all the units produced. Thus the larger the quantity, the smaller the average cost. In this case, the resulting economies of scale may not have the same implications on pricing as network effects, because when setting prices, a profit-maximizing firm will ignore the fixed costs and base its pricing decision on the marginal costs only. Without other factors at play, this type of scale economies does not have any direct impact on firms' pricing decisions.

Another important source of scale economies is learning by doing, which means that a firm becomes more efficient in its production process as more units are produced. Therefore a larger quantity results in a lower marginal cost. This creates an incentive for penetration pricing similar to the one under network effects.

Since Robinson and Lakhani (1975) there have been many studies on dynamic pricing under learning by doing, or experience effects. Robinson and Lakhani (1975) discuss a monopolist's dynamic pricing policy under experience effects and product diffusion. Using an illustrative example, they show that initial prices could be well below the initial costs, which suggests that penetration pricing can be completely justified for the sake of long-run profits.

Since learning by doing and network effects have similar implications for pricing, some researchers include learning by doing as one type of network effects (e.g. Bensaid and Lesne, 1996). However, it is still important to recognize the distinction that learning by doing reduces production costs while network effects increase product values.

6.   Empirical research

As evidenced by the large number of studies, the topic of pricing under network effects has been examined extensively in the theoretical literature. It is shown that network effects provide an incentive for firms to engage in penetration pricing. Under certain conditions an increasing price path can be optimal in both monopoly and oligopoly settings. Compared with this rich theoretical literature, empirical studies on this topic have been scarce. Thus we are still not well equipped to provide normative guidance on firms' pricing strategies in real industry settings.

On the demand side, however, there have been many empirical papers that show the existence of network effects in various markets (e.g. Nair et al., 2004 on PDAs (personal digital assistants); Clements and Ohashi, 2005 on video game consoles). These demand-side models can be extended in order to establish firms' optimal pricing strategies on the supply side.

In such an attempt, Liu (2006) studies the dynamics of pricing in the video game console market. Clearly the existence of indirect network effects provides an incentive for penetration pricing for game consoles. But due to the rapid decline in costs, this incentive does not lead to increasing console prices. Instead, we observe decreasing prices but increasing markups over time. On the other hand, consumers put different valuations on game consoles, which create an incentive for price-skimming. Based on the increasing markups, this incentive for price-skimming seems to be dominated by the competing incentive for penetration pricing due to indirect network effects.

To explain the observed price and markup patterns, Liu estimates a demand model similar to those in Nair et al. (2004) and Clements and Ohashi (2005). He then solves for the optimal pricing policies of competing console makers (i.e. Sony and Nintendo in the time period under study). It is shown that the optimal pricing policies are consistent with the observed price and markup patterns.

For empirical studies, the demand systems are relatively complicated. This often makes it infeasible to obtain analytical solutions to firms' dynamic pricing problems. As demonstrated by Liu (2006), numerical dynamic programming techniques prove useful in solving these dynamic pricing problems.

Special attention is needed on the function form of the network effects. Linear network effects are often assumed in analytical models (e.g. Bensaid and Lesne, 1996; Cabral et al., 1999; Mason, 2000; Gabszewicz and Garcia, 2005). That is, the value that a network provides increases linearly with its installed base. Although this could be a good approximation at initial stages of a product life cycle, decreasing marginal network benefits may eventually take place. For example, when the use of the telephone was less common, it was important that one million people joined the telephone network, but today it is probably not a big deal whether one million people join or quit the telephone network. Swann (2002) argues that linear network effects can only be generated under very restrictive conditions, and most communication networks exhibit decreasing marginal network benefits. Therefore it is important for future empirical work to allow for flexible specifications of network effects.

7. An illustrative example

We illustrate the issues involved in empirical studies using the following example. Assume there are M potential consumers and J competing products in a durable-goods market characterized by network effects. Each product j is sold by a single-product firm j for T time periods.

The demand for product j in period t can be written as

$$Q_{jt}(p_t, n_t, M_t)$$

where p_t is the vector of prices, n_t is the vector of network sizes, and the network size of product j is simply its cumulative unit sales:

$$n_{jt} = \sum_{\tau=1}^{t-1} Q_{j\tau}$$

M_t is the market size, or equivalently the number of consumers who have not bought any of the J products. M_t and n_t are related since

$$M_t = M - \sum_{j=1}^{J} n_{jt}$$

Naturally the demand for product j decreases with its own prices but increases with its own network sizes and the market size, i.e.

$$\frac{\partial Q_{jt}}{\partial p_{jt}} < 0, \ \ \frac{\partial Q_{jt}}{\partial n_{jt}} > 0, \ \ \frac{\partial Q_{jt}}{\partial M_t} > 0$$

Firms' current prices affect not only their current demand, but also their future demand through future network sizes and future market sizes. Therefore, when setting prices each firm will look beyond the current period and maximize the expected present value of all current and future profits:

$$E\left[\sum_{\tau=t}^{T} \delta^{\tau-t} \pi_{j\tau}\right]$$

where δ is a discount factor, and the profit function is

$$\pi_{jt} = (p_{jt} - c_{jt}) Q_{jt}$$

Although firms' pricing decisions could potentially depend on the entire history of past states and actions, for simplicity it is often assumed that firms set prices based on the current state only. Let S_t be the state vector, which consists of all the current payoff relevant variables including M_t, n_{jt} and c_{jt}. The evolution of S_t is governed by a Markov transition density $F(S_{t+1}|S_t, p_t)$ conditional on current prices.

First we consider a monopoly market with $J = 1$. Subscript j can be omitted in this case. Define the value function

$$V_t(S_t) = \max_{p_t} E\left[\sum_{\tau=t}^{T} \delta^{\tau-t} \pi_\tau(S_\tau, p_\tau)\,|\,S_t, p_t\right]$$

The optimal pricing policy can be obtained by solving the following Bellman equation

$$V_t(S_t) = \max_{p_t}\{\pi_t(S_t, p_t) + E[V_{t+1}(S_{t+1})\,|\,S_t, p_t]\}$$

Each value function $V_t(S_t)$ is associated with an optimal pricing policy $p_t(S_t)$. Usually it is infeasible to solve the dynamic pricing problem analytically, and hence numerical dynamic programming techniques need to be applied.

If the time horizon T is finite, we can start from the last time period and solve backwards in time. With an infinite horizon $T = \infty$, the form of the value function V_t does not change across time periods. Therefore the Bellman equation becomes

$$V(S) = \max_{p}\{\pi(S, p) + E[V(S')\,|\,S, p]\}$$

Starting from an initial guess of the value function, we can iterate on the Bellman equation until it converges to the final solution. Rust (1994) shows that, under fairly weak regularity conditions, the above Bellman equation has a unique solution.

Consumers are assumed to be heterogeneous so that some of them are willing to pay more than others. Suppose marginal costs remain constant over time. In the absence of network effects, in which case Q_{jt} is independent of n_t, the monopolist has incentives to set a high price initially and cut it later. Thus price-skimming may be the optimal strategy.

However, in the presence of network effects, there is a competing incentive to price low initially in order to build up the network. This incentive for penetration pricing may or may not dominate the incentives for price-skimming depending on the strength of network effects. As a result, prices can be increasing or decreasing.

To make the above discussion concrete, we consider a simple demand system. The indirect utility that a consumer i derives from a product is specified as

$$U_{it} = \alpha_i + \beta p_t + \gamma n_t^\lambda + \varepsilon_{it}$$

Here consumers differ in their intrinsic preferences toward the product according to a distribution function $F(\alpha_i)$. A consumer's individual taste, ε_{it}, follows a Type I extreme-value distribution. The outside option is normalized to have a mean utility of zero net of an individual taste. Therefore the demand function is given by

$$Q_t(p_t, n_t, M_t) = M_t \int \frac{\exp(\alpha_i + \beta p_t + \gamma n_t^\lambda)}{1 + \exp(\alpha_i + \beta p_t + \gamma n_t^\lambda)} dF(\alpha_i)$$

To solve for the optimal pricing policy, we assume a potential market size of 200 and a discrete distribution on α_i: 10 percent of consumers have $\alpha = -2$ and the rest have $\alpha = -5$. For other parameters we assume $\beta = -0.02$, $\gamma = 1$, $\lambda = 0.3$ and a discount factor of 0.995. These parameter values are consistent with the estimates for the Palm Vx PDA in Nair et al. (2004).

After solving for the optimal pricing policy with a finite horizon of 24 time periods, we simulate the market evolution and plot the price path in Figure 20.2. It indicates an increasing price path under network effects. But without network effects, we would see decreasing prices over time.

Now consider an oligopoly market in which each firm's pricing decision has to take into account the pricing policies of other firms. We need to solve the dynamic pricing game for the equilibrium pricing policies. The equilibrium concept often in use is the Markov-perfect equilibrium (MPE) in pure strategies. Maskin and Tirole (2001) provide a concise treatment of the MPE concept.

Given other firms' pricing policies, a particular firm's pricing policy can be obtained by following a similar algorithm to the one used for a monopoly market. We can then iterate through all firms' pricing policies until convergence. Unlike the single-agent dynamic optimization problem, there is no general result that guarantees the existence and uniqueness of an equilibrium. In practice, the convergence of the solution algorithm confirms the existence, and starting the algorithm from different initial values may help find evidence of multiple equilibria.

In an oligopoly market, incentives for both price-skimming and penetration pricing
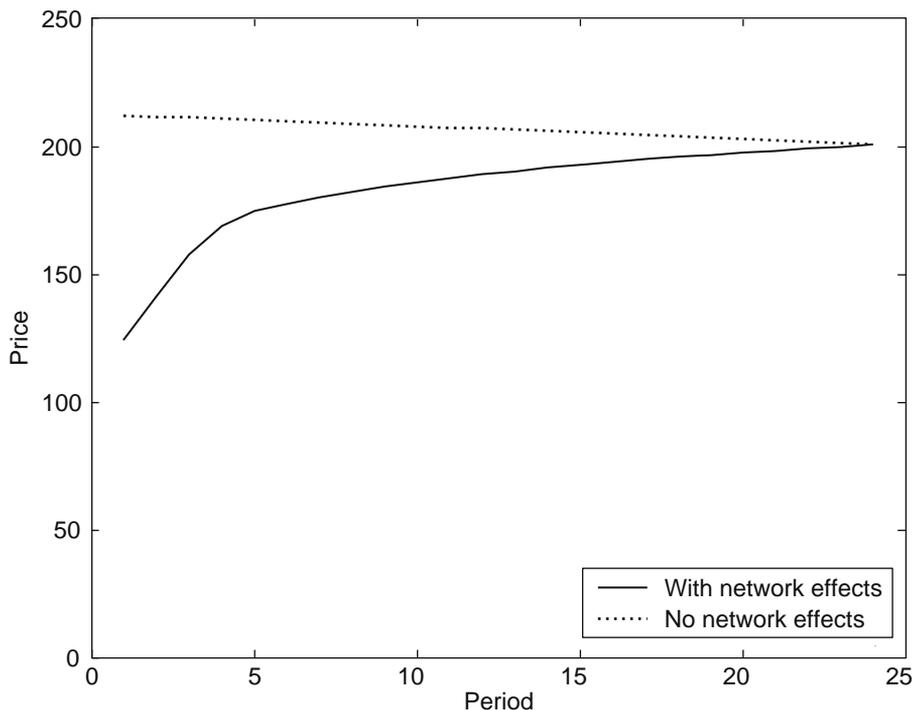
Figure 20.2   Simulated prices with and without network effects

still exist, just as in a monopoly market. Competition may push initial prices lower than those under monopoly. But as we explained previously, a monopolist may expect more profits in future periods than oligopolists, and hence may be willing to cut initial prices even deeper. Therefore an oligopoly does not necessarily lead to lower initial prices.

If the market exhibits learning by doing, or experience effects, marginal costs will decline as more units are produced. This adds to the incentives for penetration pricing since a low initial price brings the additional benefit of reducing unit production costs. It should be noted that, despite stronger incentives for penetration pricing, an increasing price path does not become more likely because costs are declining. Therefore it might be useful to examine the unit markups. Even if prices decrease, the incentives for penetration pricing could still be revealed by increasing markups.

In order to fit this model to empirical data, generally there are two sets of parameters to be estimated. On the demand side, there may be parameters in the demand function Q_{jt}. On the supply side, there may be parameters in the cost function c_{jt}. A joint estimation of demand and supply is attractive in terms of efficiency. But, recognizing the computational burden in solving the dynamic pricing game, we may resort to a two-step approach. In the first step, we can use data on quantities, prices and other covariates to estimate the demand parameters. In the second step, we can use the optimal pricing model to estimate the parameters on the supply side.

It should be mentioned that, if the costs are estimated in this way, implicitly firms are assumed to set prices optimally. This may or may not be an issue depending on the purpose

of the study. If we want to analyze firms' current pricing strategies and provide guidance on how firms should set prices, then optimality assumption is not appropriate and cost estimates should come from other sources. In such cases a two-step approach is required.

8.   Conclusions and future research

Firms' pricing strategies are intrinsically dynamic under network effects. Various issues on dynamic pricing of network goods have been examined carefully by a number of theoretical studies. In particular, the incentive for penetration pricing is emphasized. This literature is closely related to the literatures on two-sided markets, switching costs and economies of scale.

Due to the asymmetry between a rich theoretical literature and a limited empirical one, further empirical research might be fruitful in this area. In addition, there have been abundant examples of new products characterized by network effects, such as online gaming (e.g. Xbox Live), instant messaging software (e.g. AOL Instant Messenger, MSN Messenger, Yahoo! Messenger), etc. which provide exciting markets and issues for empirical studies.

As we have mentioned in the previous section, network effects are often assumed to be linear in network sizes. With empirical data, we can allow for a flexible specification of the network effect and uncover any decreasing marginal network benefits. A nonlinear network effect could affect firms' pricing policies differently from a linear effect.

In most network industries, firms' pricing decisions are affected by certain other factors besides network effects. The incentive for penetration pricing induced by network effects can be either strengthened or weakened by other factors. For example, learning by doing could provide a similar incentive for penetration pricing to network effects, while consumers' heterogeneous valuations could provide a competing incentive for price-skimming. Empirically we can estimate the magnitude of such factors and identify the effect of each on firms' pricing policies.

Consumer expectations play an important role in the diffusion of network products. Consumers' adoption decisions may depend on their expectations on future prices and network sizes. Different assumptions can be made on these expectations, ranging from completely myopic to perfectly forward looking. In an empirical model, we often rely on numerical techniques to solve firms' dynamic pricing problems. If consumers are perfectly forward looking, their expectations will be consistent with future states of the market in equilibrium. Such a model could be challenging to solve. However, Dubé et al. (2008) have made significant progress on this front recently.

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